

Geomechanics

LECTURE 10

GEOMECHANICAL BEHAVIOUR OF UNSATURATED GEOMATERIALS

PROF. LYESSE LALOU

Laboratory of soil mechanics - Fall 2025

Swiss concept for HLW disposal (Nagra)

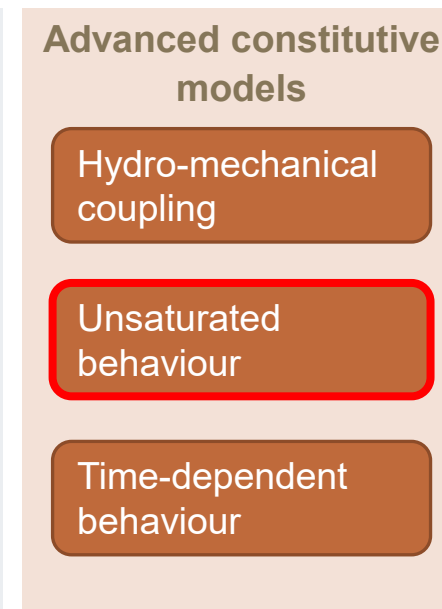
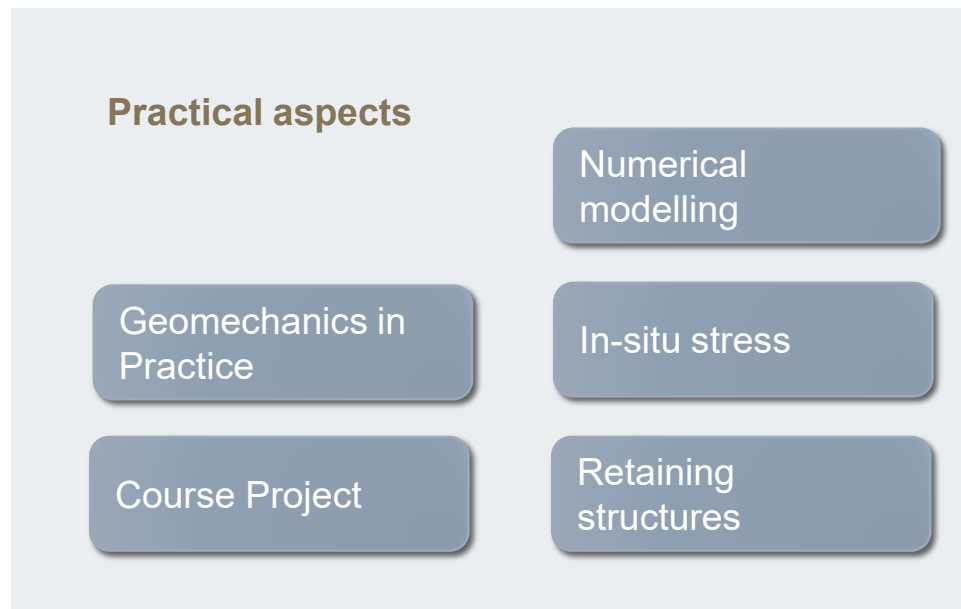
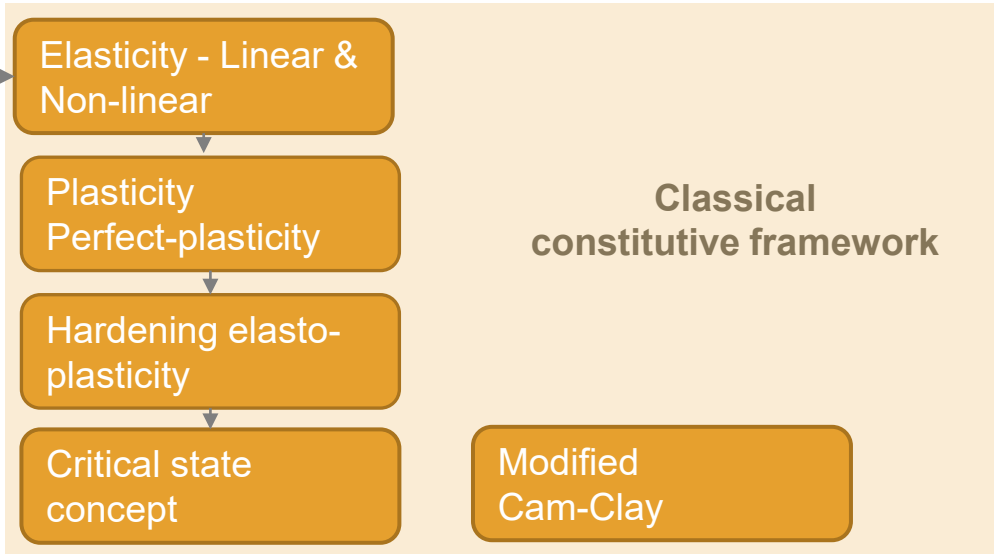


Access the QUIZ



<https://etc.ch/NLsn>

Basic concepts



Topics

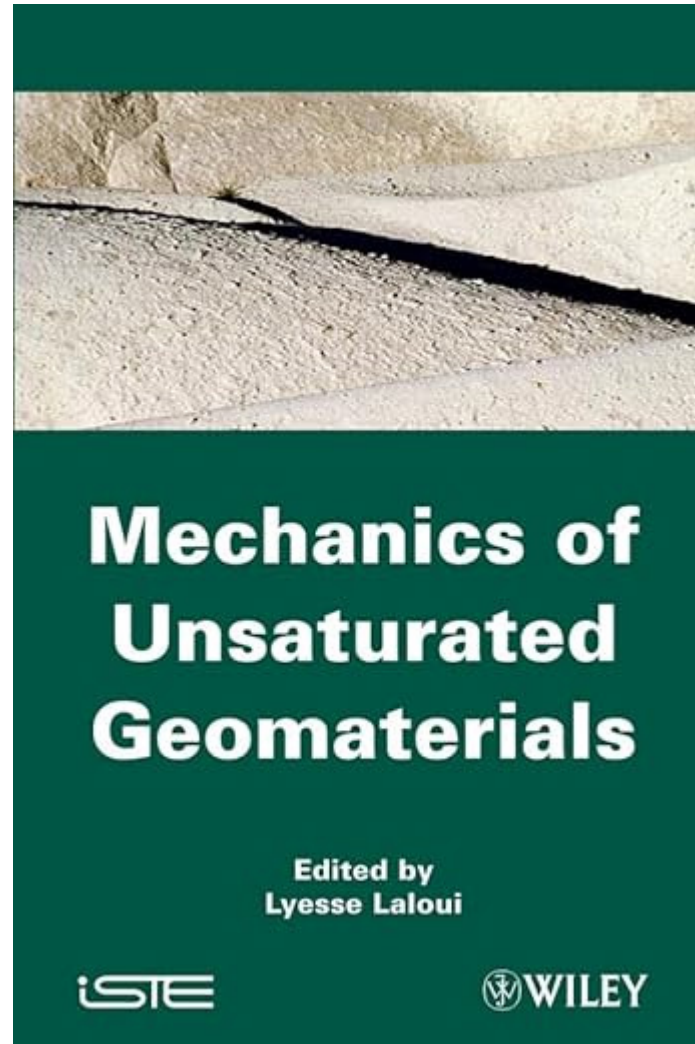
- Basic concepts
- Hydraulic properties of geomaterials under unsaturated conditions
- Mechanical behaviour of unsaturated geomaterials
- Conclusion

Basic concepts

INTRODUCTION

THE DEFINITION OF SUCTION

Useful reference

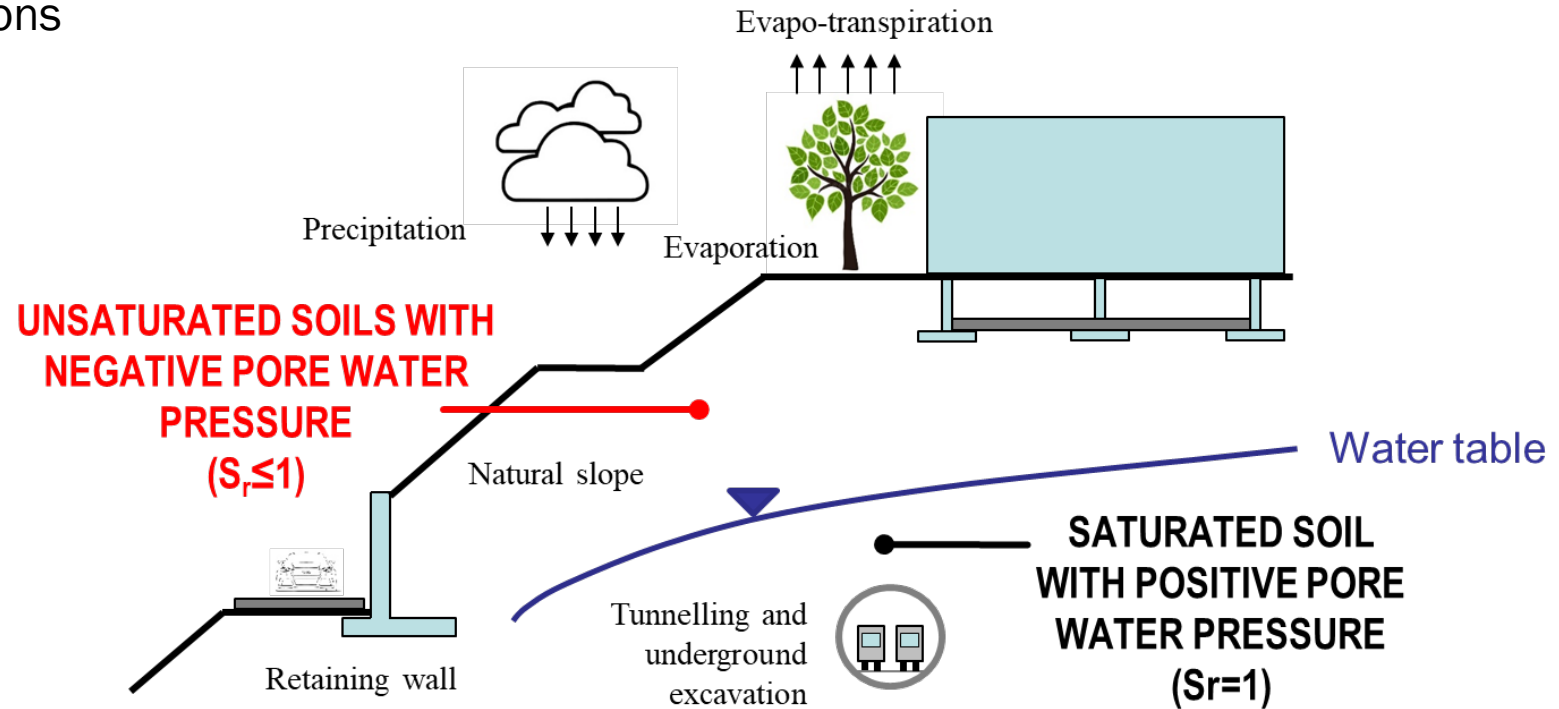


Basic concepts unsaturated soils

Saturated and unsaturated states in practice

- Embankments and tunnels
- Road foundations
- At ground surface (infiltration and evaporation)
- Geoenvironmental engineering applications (nuclear and domestic waste storage)

Hydro-mechanical properties of the soil need to be characterized in saturated and in unsaturated conditions



Above the water table the pore water pressure is negative, and the soil can become unsaturated

Basic concepts unsaturated soils

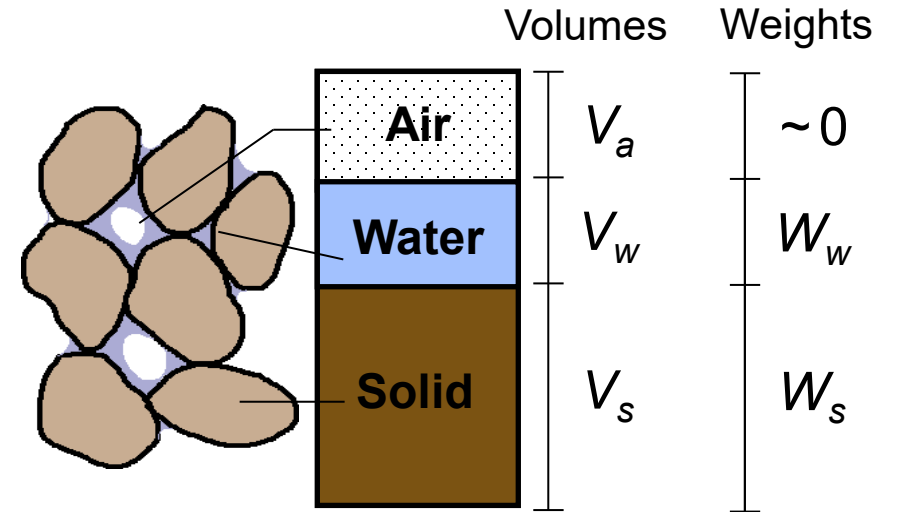
Unsaturated soils

3 phases

- Gas phase (air)
- Liquid phase (water)
- Solid phase (solid particles)

3 stress variables

- σ total mechanical stress
- p_w porewater pressure
- p_a pore-air pressure



Three-phase description

Many ways to quantify the amount of water

- Gravimetric water content
- Degree of saturation
- Volumetric water content
- Water ratio

$$w = W_w/W_s$$

$$S_r = V_w/V_v$$

$$q = V_w/V$$

$$e_w = V_w/V_s$$

- Volume of voids $V_v = V_a + V_w$
- Total volume $V = V_v + V_s$
- Porosity $n = V_v/V$
- Void ratio $e = V_v/V_s$

Basic concepts unsaturated soils

Complex interaction between water, air and solid phases

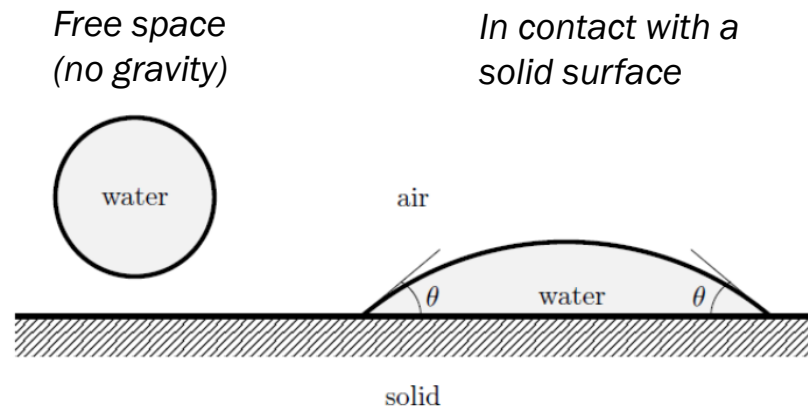
Surface tension, T :

- energy per unit area of the surface between phases
- energy penalty of breaking the intermolecular interactions between the two phases, and instead creating an interface between them

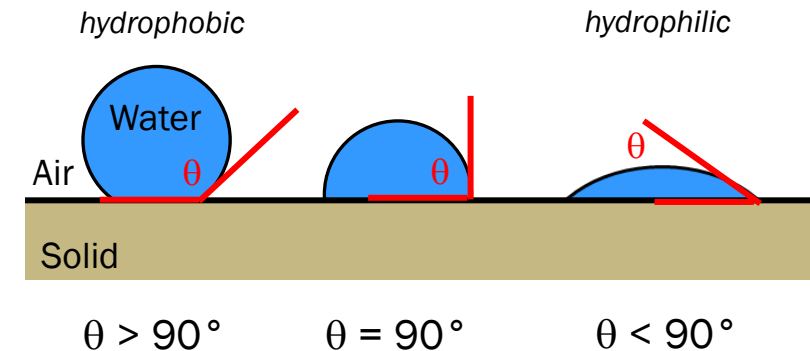
Contact angle, θ :

- angle between two fluids on a solid surface
- expresses the wettability of the solid
- measured through the denser phase, usually water

Fluid arrangements



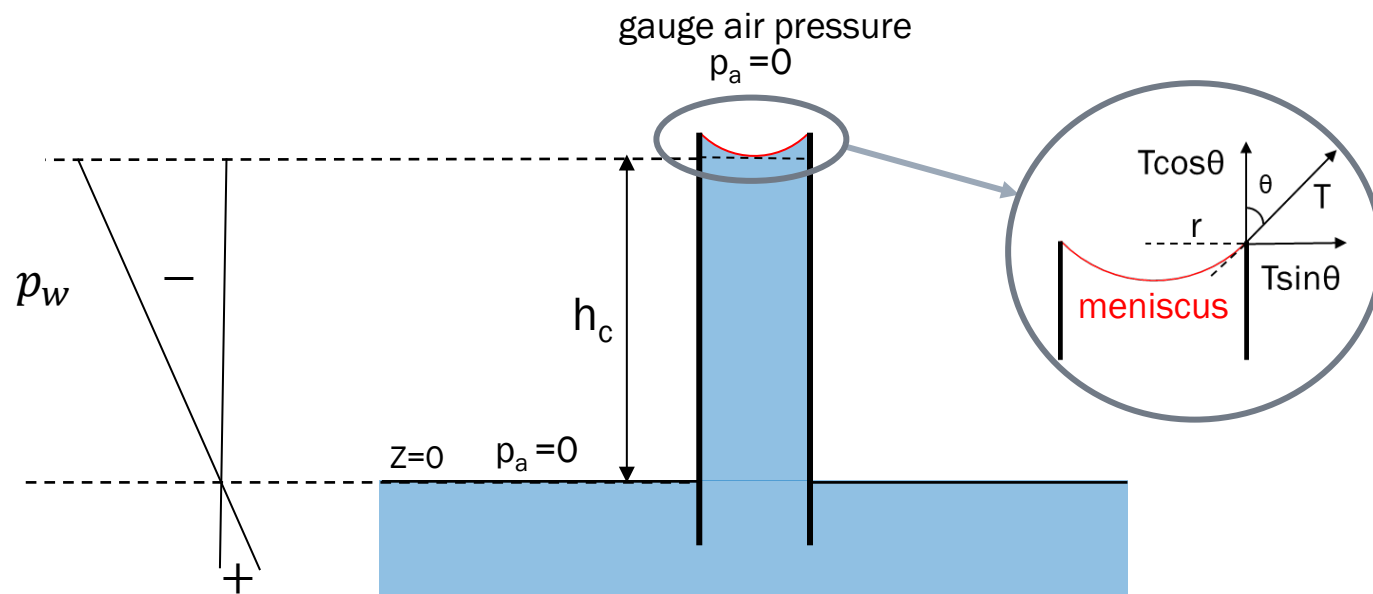
Representative values
 Air/water: 72 mN/m
 Oil/water: 50 mN/m
 Air/oil: 20 mN/m



Basic concepts unsaturated soils

Complex interaction between water, air and solid phases

- The interface air-water near a solid surface has a curvature due to **surface tension**
- The water in the voids is similar to the water in a capillary tube.



T: Surface tension (force per unit of length)
 θ : contact angle (it depends on the characteristics of the liquid and the solid)

Vertical equilibrium of the water column

$$\gamma_w h_c \pi r^2 = 2\pi r T \cos\theta$$



$$h_c = \frac{2\pi r T \cos\theta}{\gamma_w \pi r^2} = \frac{2T \cos\theta}{\gamma_w r}$$

$$p_w = -\gamma_w h_c = -\frac{2T \cos\theta}{r}$$

The gauge water pressure is negative (p_w)

Young-Laplace equation

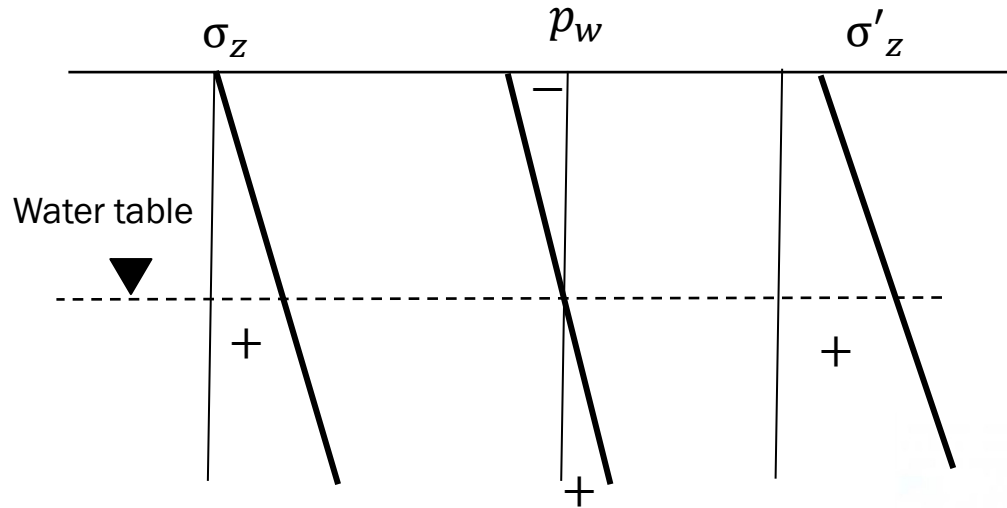
$$s = p_a - p_w = \frac{4T \cos\theta}{D}$$

Suction s

$$D = 2r$$

Basic concepts unsaturated soils

A **negative pore water pressure** allows having **non-zero effective stresses** even at a zero depth.



The **Terzaghi's effective stress** can be considered still valid if the soil remains saturated even with negative pore water pressure

$$\sigma'_z = \sigma_z - p_w$$



The **shear strength** can be higher than zero even for a granular non-cemented soil at zero depth

$$\tau_f = c' + \sigma'_n \tan \phi'$$

The sand castle is an example of how the negative pore water pressure in nearly saturated states affect the mechanical properties of soils.



Basic concepts unsaturated soils

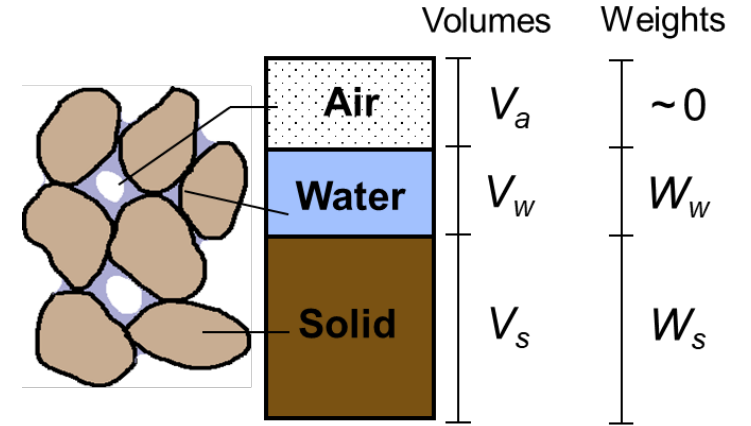
Unsaturated soils

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3 stress variables

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- p_w porewater pressure
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Three-phase description

A complete description of the rheological behaviour of unsaturated soils requires

- Soil water retention behaviour
- Stress-strain behaviour

How S_r changes for a given suction ?

How the soil deforms?

Stress-strain variables for constitutive modelling of unsaturated soils

Stress variables

$$\begin{pmatrix} \sigma' \\ s \end{pmatrix}$$

Work-conjugate variables

$$\begin{pmatrix} \varepsilon \\ S_r \end{pmatrix}$$

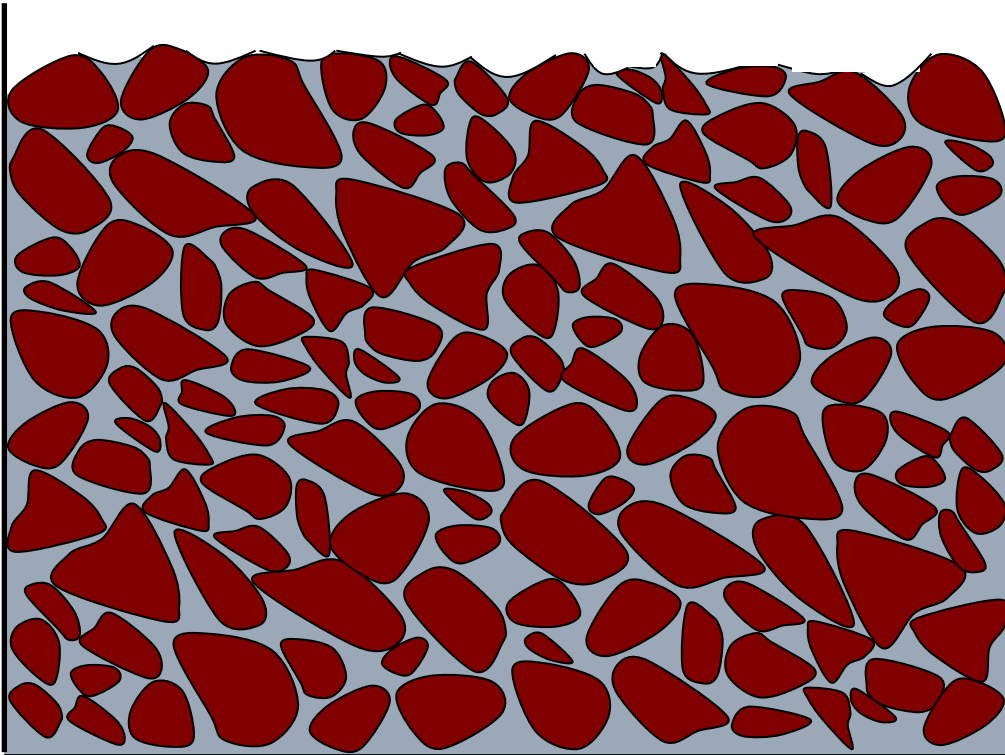
Hydraulic properties of geomaterials under unsaturated conditions

THE WATER RETENTION CURVE

THE PERMEABILITY

The water retention curve

SATURATED STATE

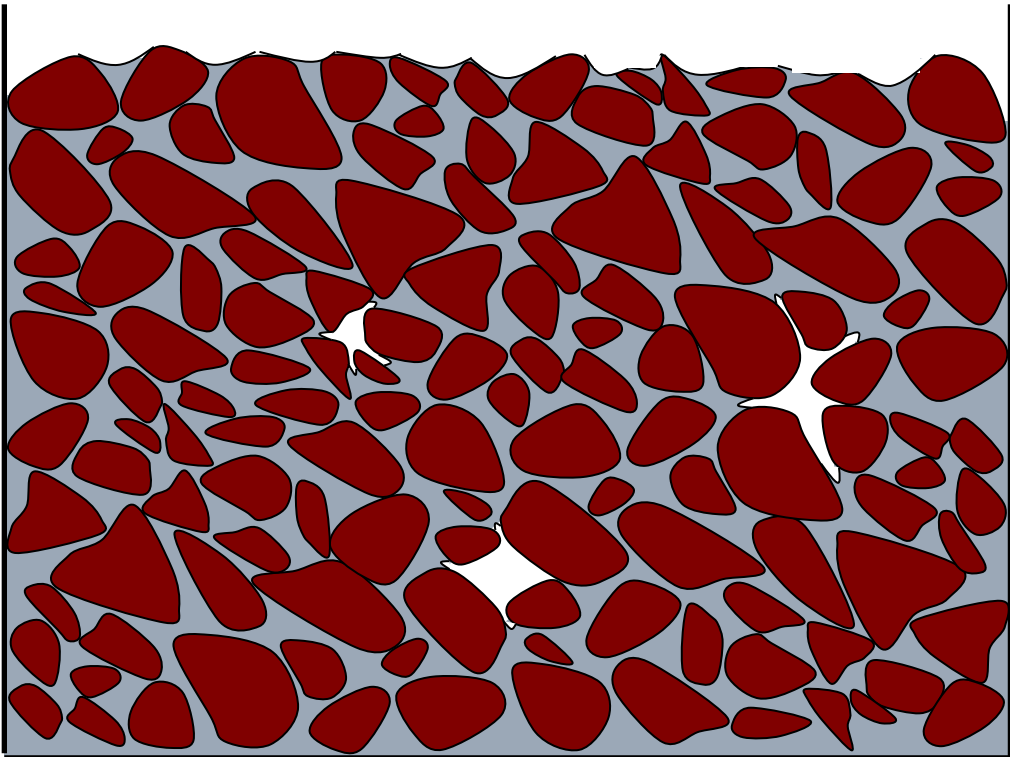


An initially saturated geomaterial is subjected to evaporation

- **Menisci** at the surface start to appear
- The pore water pressure decreases and the suction becomes higher than 0, $s > 0$
- The soil is still saturated, $S_r = 1$

The water retention curve

QUASI - SATURATED STATE

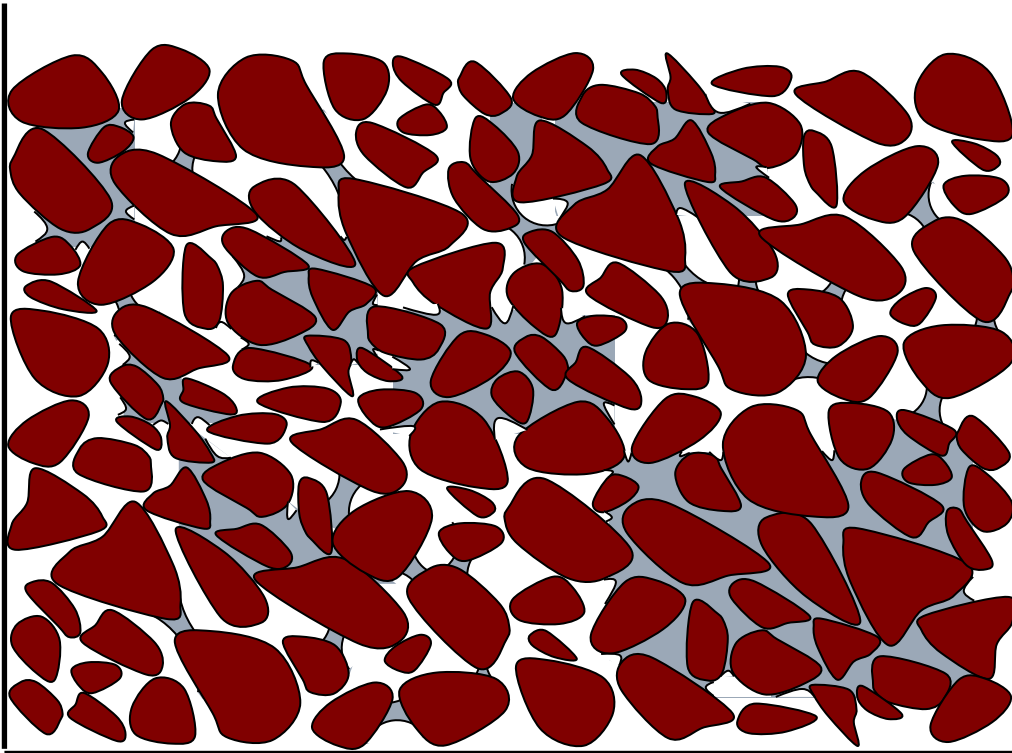


Indicative values of the degree of saturation
 $0.85-0.90 < S_r < 1$

- The pore water pressure continues decreasing and the suction continue increasing, $s > 0$
- More menisci appear, air cavities in the larger pores expand due to negative porewater pressure
- **Gas** phase is **discontinuous**, **liquid** phase is **continuous**
- The degree of saturation decreases, $S_r < 1$

The water retention curve

PARTIALLY SATURATED STATE

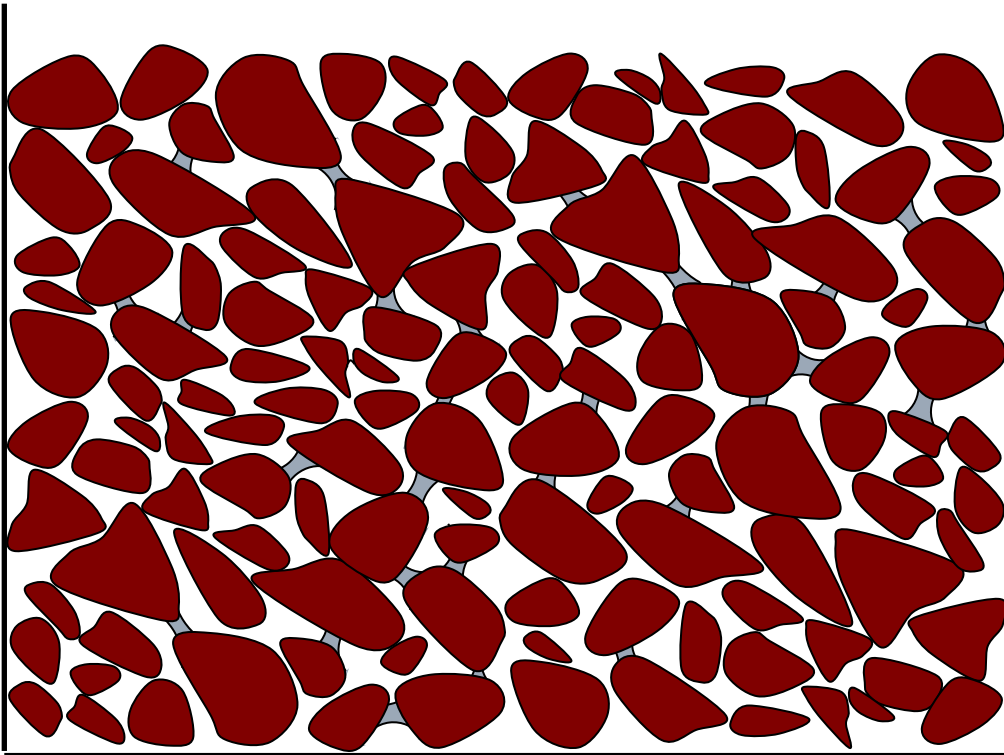


Indicative values of the degree of saturation
 $0.1 < S_r < 0.85-0.90$

- More menisci appear, smaller pores start to desaturate;
- At the interface, the maximum curvature of the menisci is reached, and the air enters
- The pore water pressure continues decreasing and the suction continues increasing, $s > 0$
- **Gas** phase is **continuous**, **liquid** phase is **continuous**
- The degree of saturation decreases, $S_r < 1$

The water retention curve

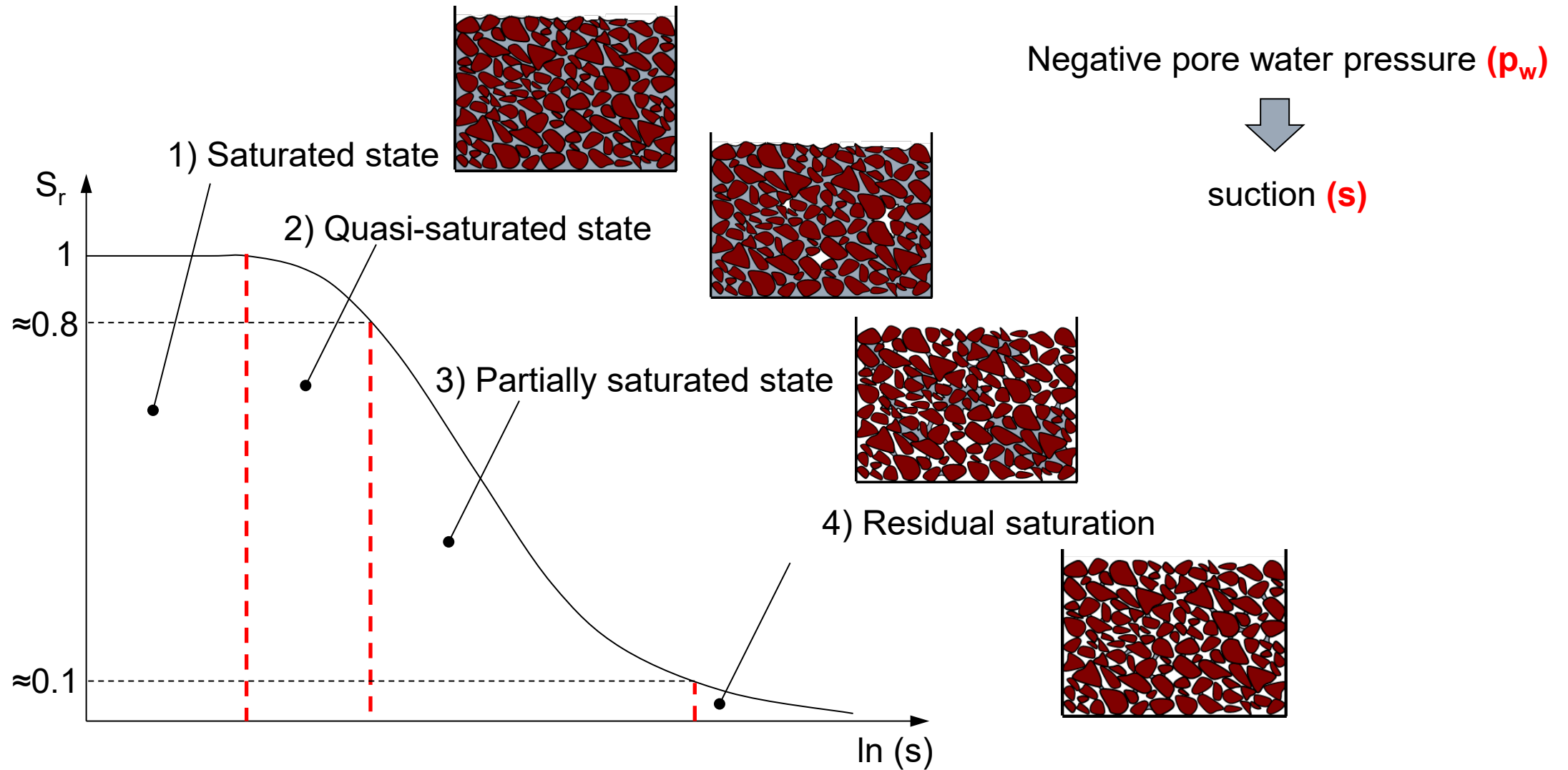
RESIDUAL STATE



Indicative values of the degree of saturation
 $S_r < 0.1$

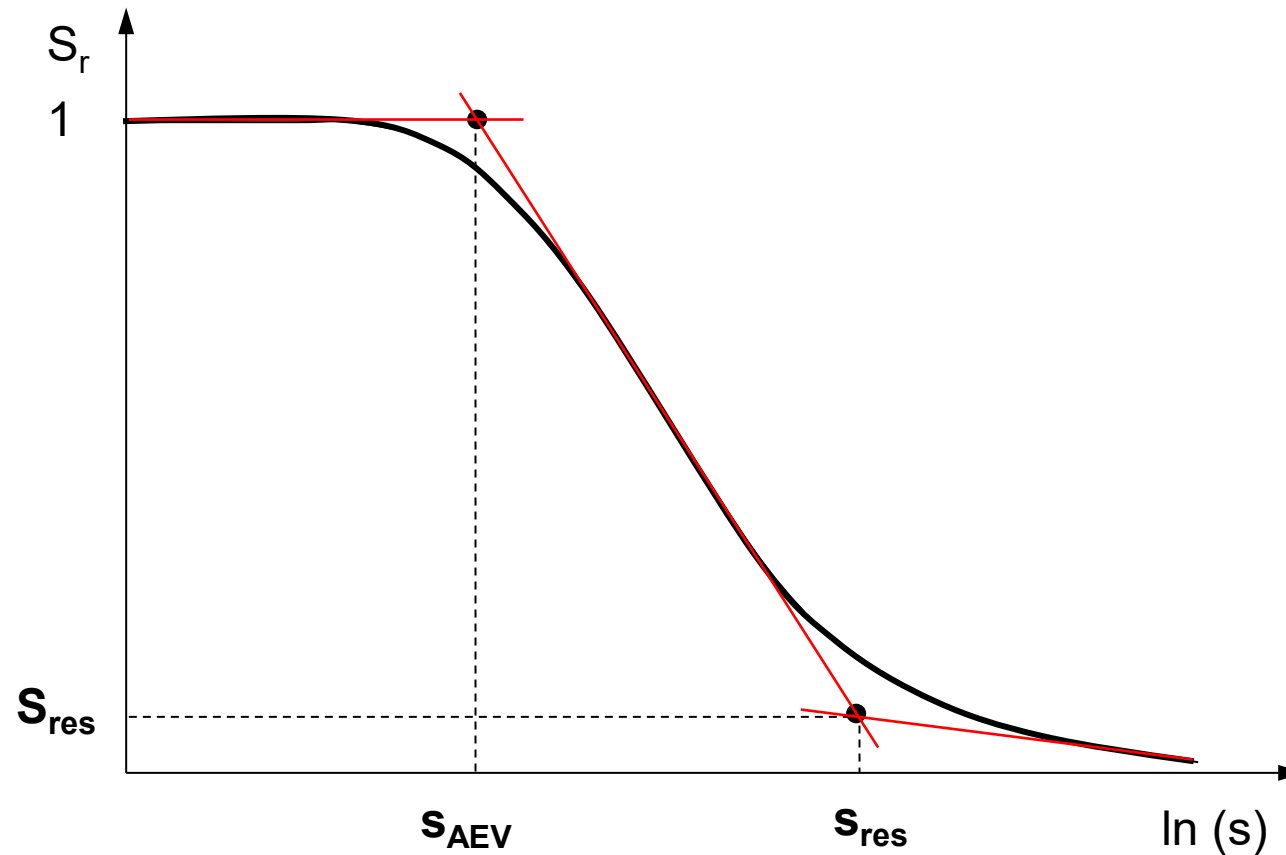
- There is water in some isolated pores
- The pore water pressure continues decreasing and the suction continues increasing, $s > 0$
- **Gas** phase is **continuous**, **liquid** phase is **discontinuous**
- The degree of saturation does not decrease significantly

The water retention curve



The water retention curve

The relationship between the degree of saturation and the suction is called “**water retention curve**”. It expresses the capacity of the soil to retain water



Key parameters

s_{AEV} = air entry value

s_{res} = residual suction

S_{res} = residual degree of saturation

The water retention curve

THE AIR ENTRY VALUE

- Value of suction from which the **soil start to desaturate**
- **Function of the pores size:** a lower suction is needed to desaturate a soil with larger pores

Young-Laplace equation

$$s = p_a - p_w = \frac{4T \cos \theta}{D}$$

D: pore diameter for different geomaterials

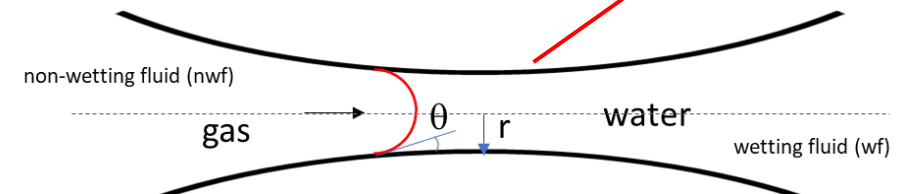
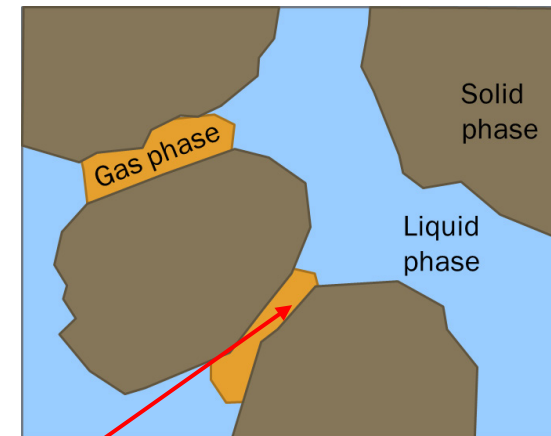
Values assumed for the other parameters

Water-air surface tension, $T = 0.072 \text{ N/m}$

Water contact angle, $\theta = 0^\circ$

Order of magnitude of the air entry value depending on the characteristic pore size:

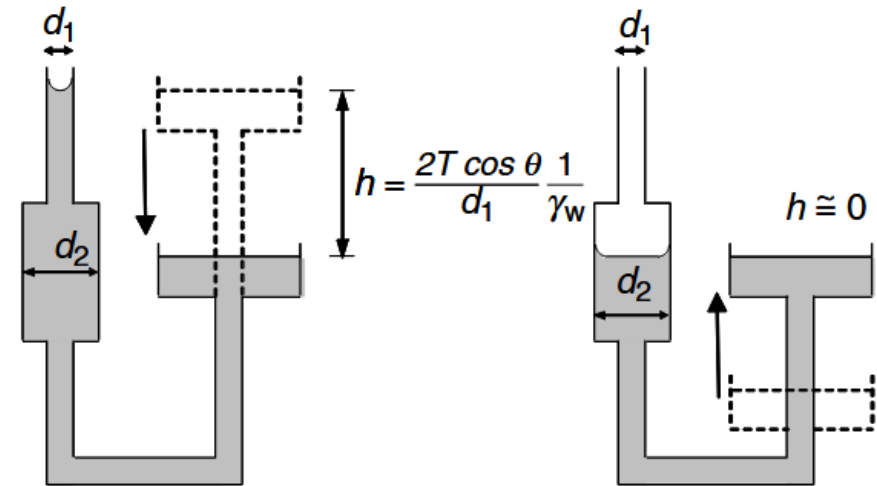
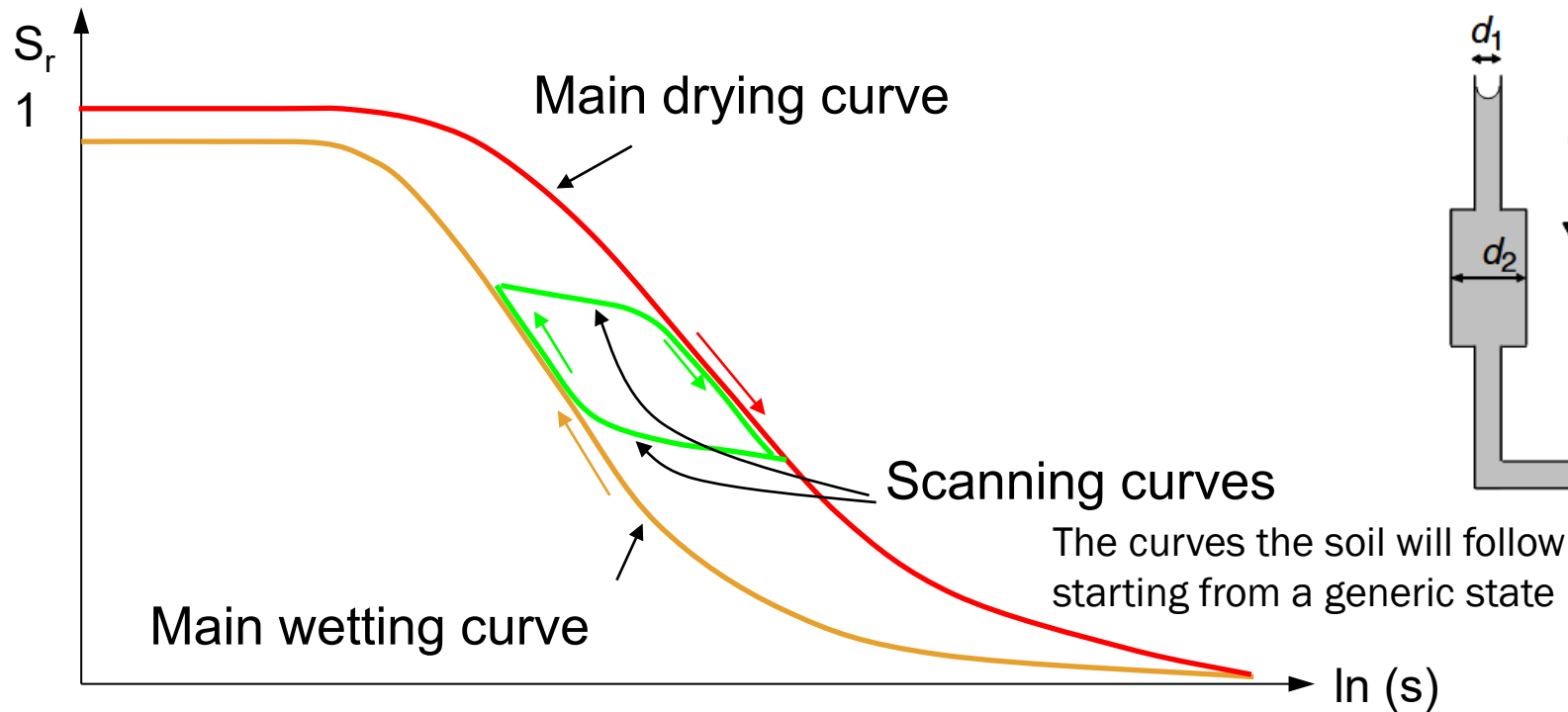
sand ($d_{\max} = 30 \text{ } \mu\text{m}$)	silt ($d_{\max} = 3 \text{ } \mu\text{m}$)	clay ($d_{\max} = 0.3 \text{ } \mu\text{m}$)
9.6 kPa	96 kPa	960 kPa



The water retention curve

DRYING AND WETTING PROCESSES – HYSTERETIC BEHAVIOR

- Increase in suction is called “drying process”
- Decrease in suction is called “wetting process”
- The s - S_r relationship depends on the alternation of these processes



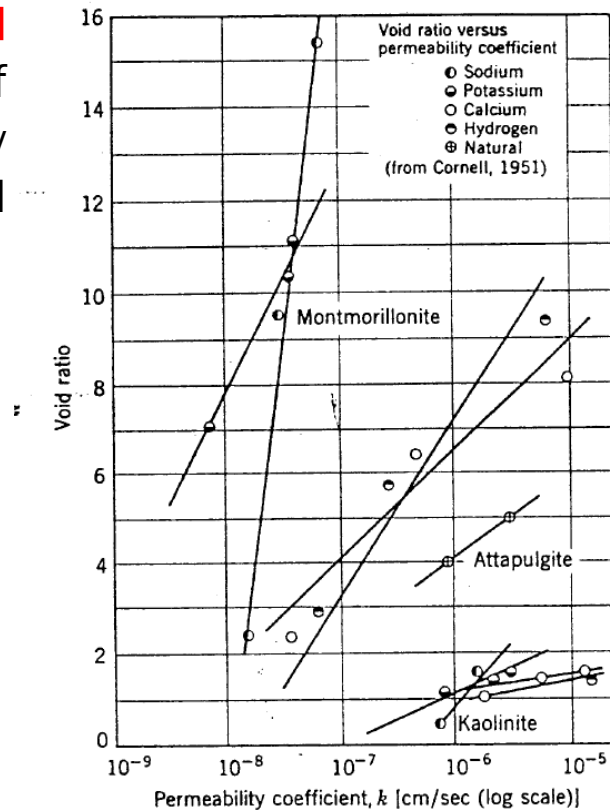
Laloui (2013)

The permeability

The **coefficient of permeability** is the coefficient of proportionality between the flow rate and the hydraulic head gradient. **Darcy's law** (for the x-direction) gives:

$$v_{w,x} = -k_w \frac{\partial h_w}{\partial x}$$

For a given **saturated soil**, the coefficient of permeability mainly depends on the void ratio

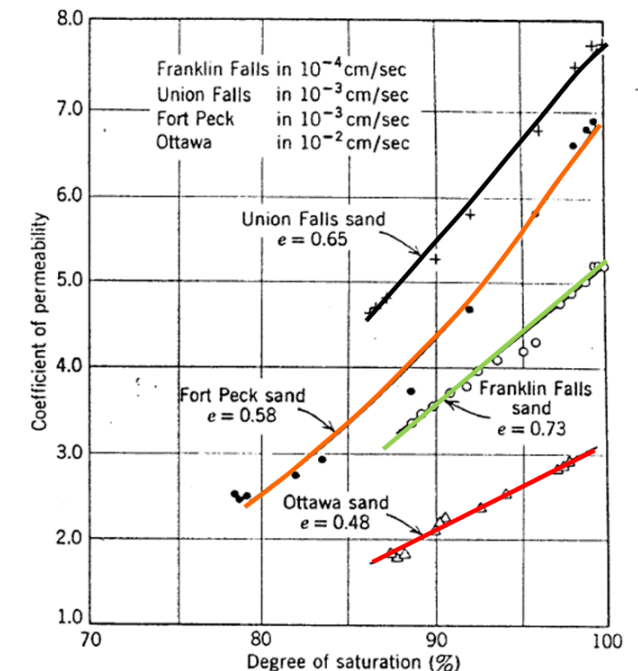


Lambe and Whitman, 1969

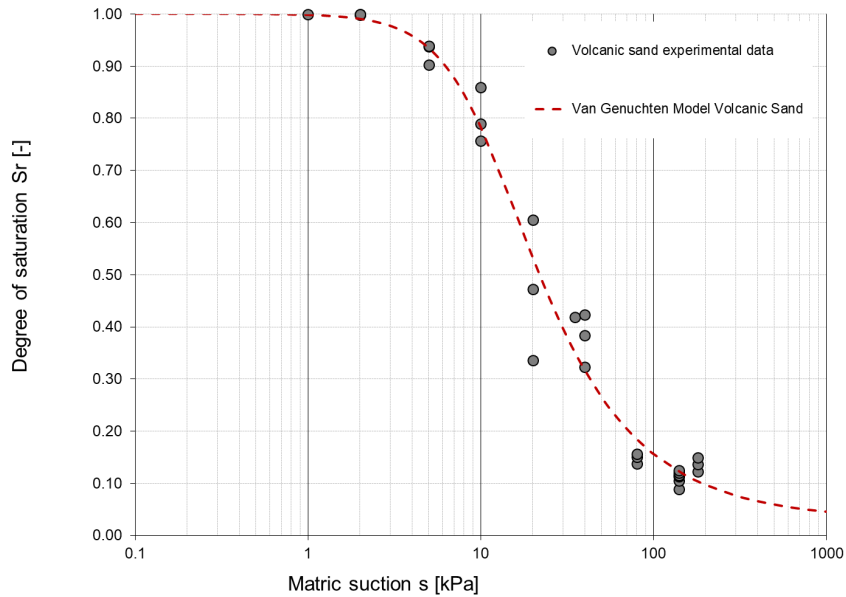
For an **unsaturated soil**, the **Darcy's law is still valid** but the coefficient of permeability depends on both the water content (or degree of saturation) and the void ratio.

At higher degree of saturation, more connected pores are used by water to flow

For a given void ratio, the higher the degree of saturation, the higher is the permeability coefficient



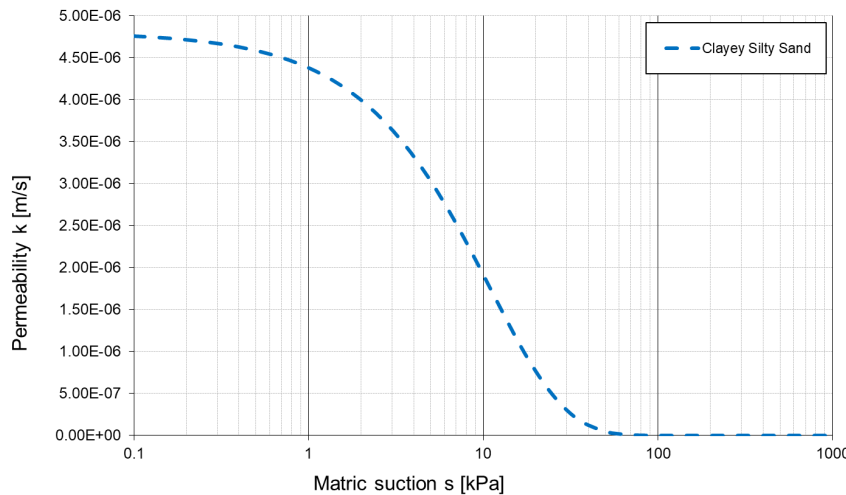
WRC and permeability



Van Genuchten, M., 1980

$$S_r = \left\{ \frac{1}{1 + [\alpha(p_a - p_w)]^n} \right\}^m$$

α, n, m fitting parameters



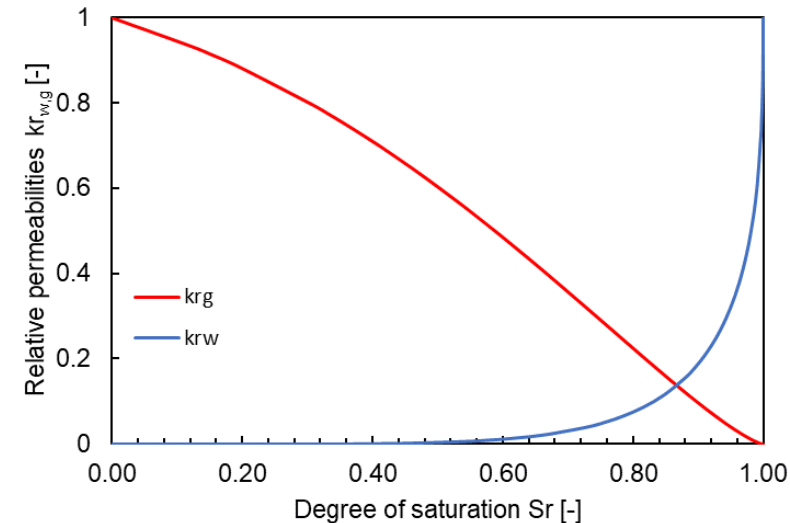
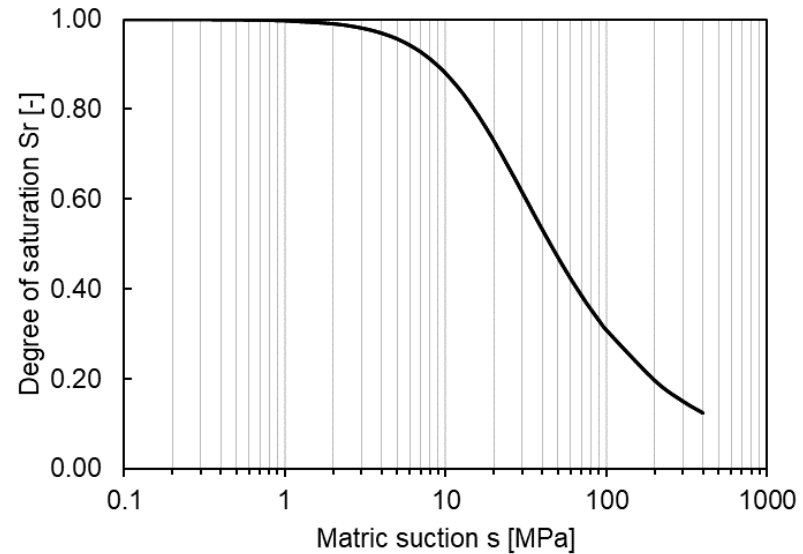
The evolution of the coefficient of permeability with suction can be described for example by using the Gardner's model (1958).

Gardner's model

$$k = k_{sat} e^{-\alpha(p_a - p_w)}$$

α fitting parameter

WRC and permeability



Van Genuchten, M., 1980

$$S_r = \left\{ \frac{1}{1 + [\alpha(p_a - p_w)]^n} \right\}^m$$

α , n , m fitting parameters

Mualem, Y. 1976

$$k_{rw} = \sqrt{S_r} \cdot \left[1 - \left(1 - S_r^{\frac{1}{m}} \right)^m \right]^2$$

$$k_{rg} = \sqrt{1 - S_r} \cdot \left[1 - S_r^{\frac{1}{m}} \right]^{2m}$$

m fitting parameter

Generalized Darcy's law for partially saturated porous media
Flux for a given fluid i , \mathbf{q}_i

$$\mathbf{q}_i = -\frac{\mathbf{k}_i}{\mu_i} (\nabla p_i + \rho_i \mathbf{g})$$

$$\mathbf{k}_i = \mathbf{K} k_{r,i}(S_r)$$

\mathbf{k} : apparent permeability
 μ : dynamic viscosity
 p : pressure
 ρ : density
 \mathbf{g} : gravity

The intrinsic permeability \mathbf{K} , is scaled by the relative permeability of the considered fluid i : $k_{r,i} \text{ fct}(S_{r,i})$

Mechanical behaviour of unsaturated geomaterials

NET STRESS AND GENERALIZED EFFECTIVE STRESS

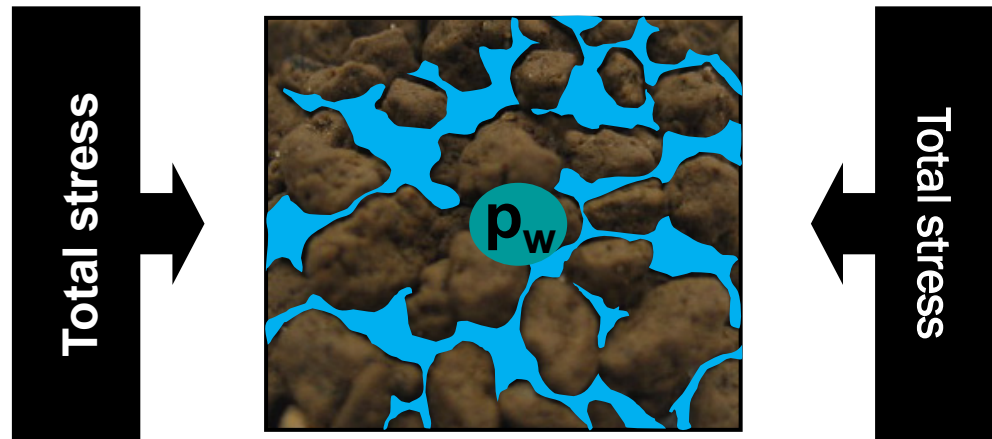
VOLUMETRIC BEHAVIOUR AND WETTING COLLAPSE

SHEAR STRENGTH

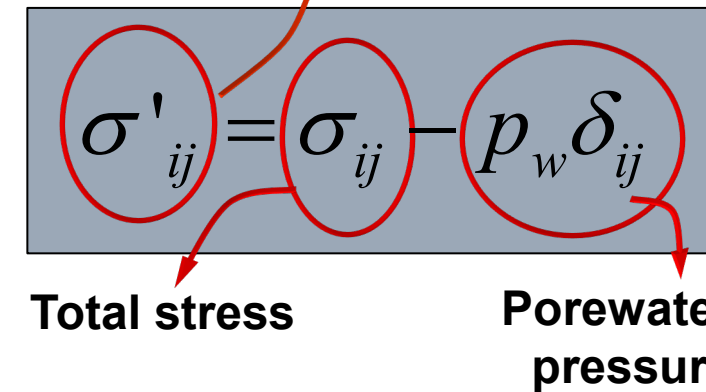
SWELLING SOILS & DESICCATION CRACKS

Suction, net stress and effective stress

We have seen that in the case of **saturated specimens** with incompressible water and solid grains the effective stress is



Terzaghi's effective stress (1936):



SUCTION

The difference between the pressure of the air and the pressure of the water

$$s = p_a - p_w = \frac{2T \cos \theta}{r} \quad s \delta_{ij} \quad \text{suction stress tensor}$$

NET STRESS

The difference between the external stress and the air pressure

$$\sigma_{net,ij} = \sigma_{ij} - p_a \delta_{ij} \quad \text{net stress tensor}$$

Suction, net stress and effective stress

If the porewater pressure is negative (suction > 0), two different situations must be distinguished:

Quasi-saturated state $s < s_{AEV}$ ($S_r = 1$)

Even if the pore water pressure is negative, the soil remains close to saturated

$$\sigma'_{ij} = \sigma_{ij} - p_w \delta_{ij} \quad \text{It is still valid}$$

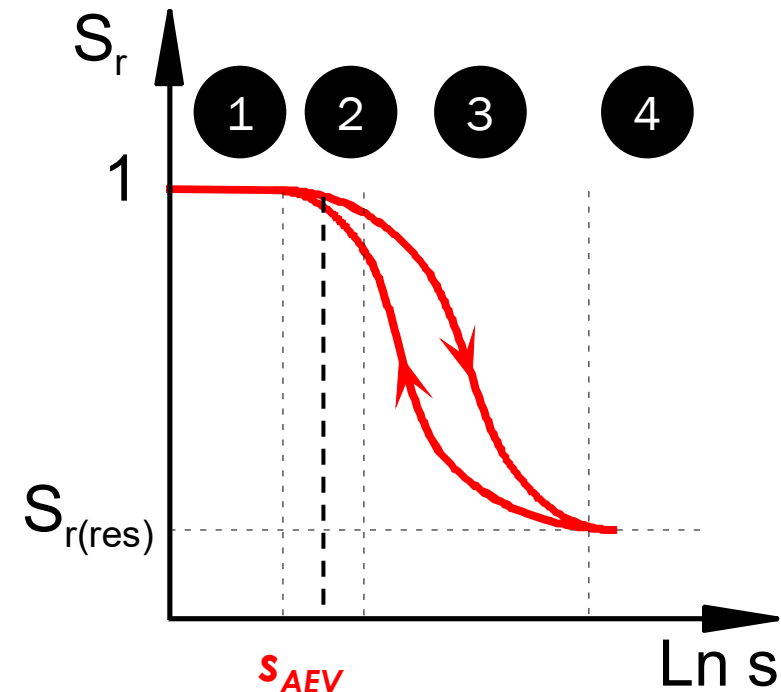
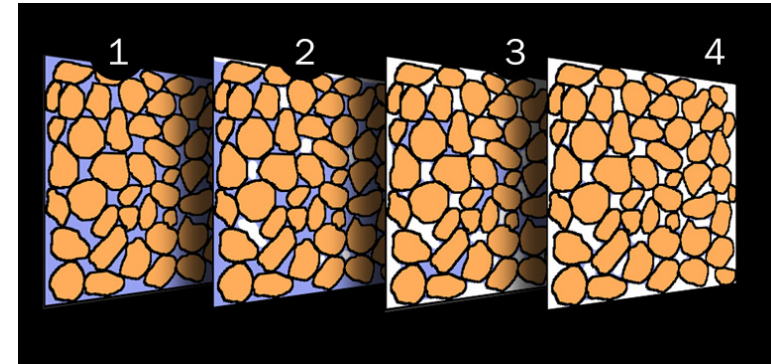
$$\rightarrow \sigma'_{ij} = \sigma_{ij} + |p_w| \delta_{ij}$$

Partially saturated state $s > s_{AEV}$ ($S_r < 1$)

The pore water pressure is negative and the soil is partially saturated

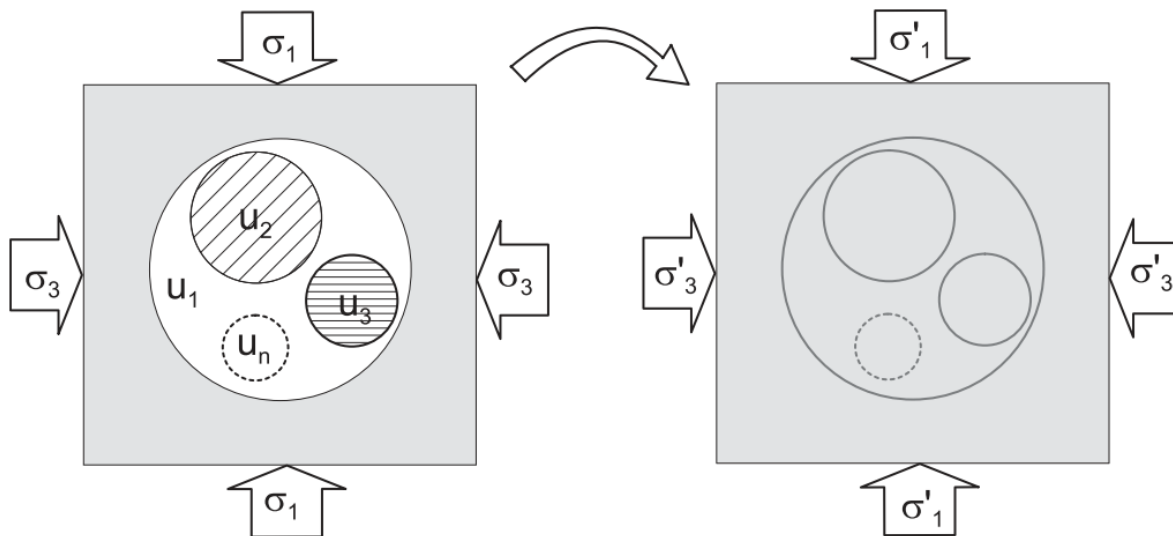
The effective stress concept needs to be generalized:

$$\rightarrow \sigma'_{ij} = \sigma_{ij} - p_{equivalent} \delta_{ij}$$



Suction, net stress and effective stress

The effective stress with more than one fluid



Conversion of a multiphase and multi stress medium in an equivalent medium

For many geomaterials the following general expression of effective stress is valid

$$\sigma'_{ij} = \sigma_{ij} - \sum_{\beta=1}^i \alpha_{\beta} p_{\beta} \delta_{ij}$$

σ_{ij} : total stress

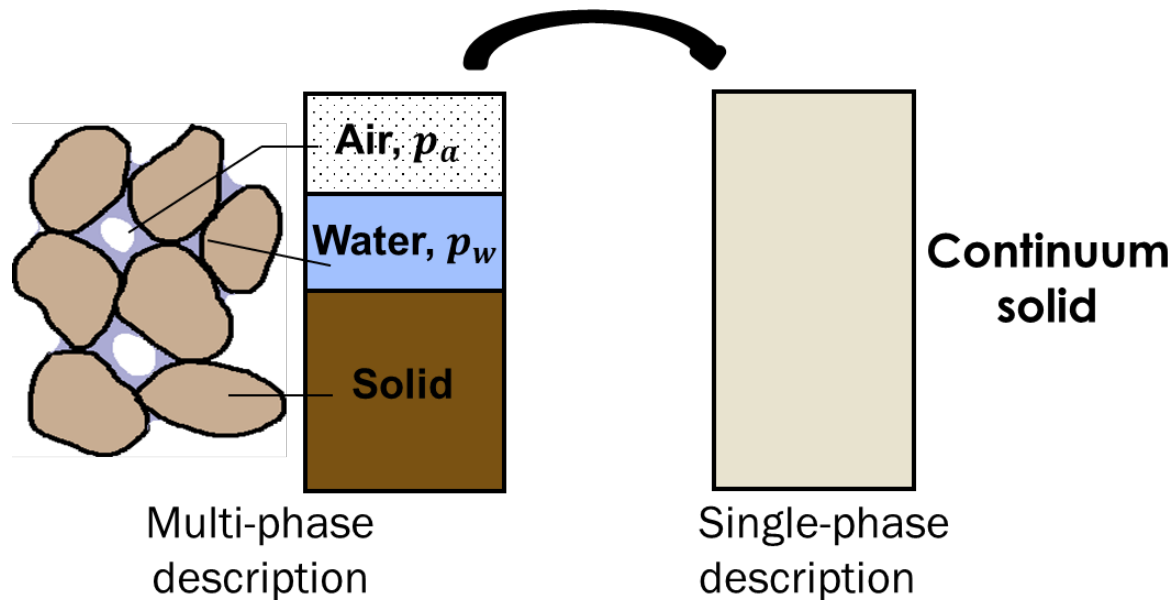
β : generic fluid

α_{β} : scaling factor of the fluid β

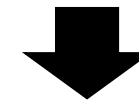
p_{β} : pressure of the fluid β

Suction, net stress and effective stress

The effective stress in the specific case of unsaturated soils



$$\sigma'_{ij} = \sigma_{ij} - \sum_{\beta=1}^2 \alpha_{\beta} p_{\beta} \delta_{ij}$$



$$\sigma'_{ij} = \sigma_{ij} - (1 - \chi) p_a \delta_{ij} - \chi p_w \delta_{ij}$$

$$\sigma'_{ij} = (\sigma_{ij} - p_a \delta_{ij}) + \chi (p_a - p_w) \delta_{ij}$$

$$\sigma'_{ij} = \sigma_{net,ij} + \chi s \delta_{ij}$$

Net stress tensor Suction stress tensor

with $\chi = f(S_r)$

if $\chi = S_r$ $\sigma'_{ij} = \sigma_{net,ij} + S_r \cdot s \delta_{ij}$

Stress-strain variables

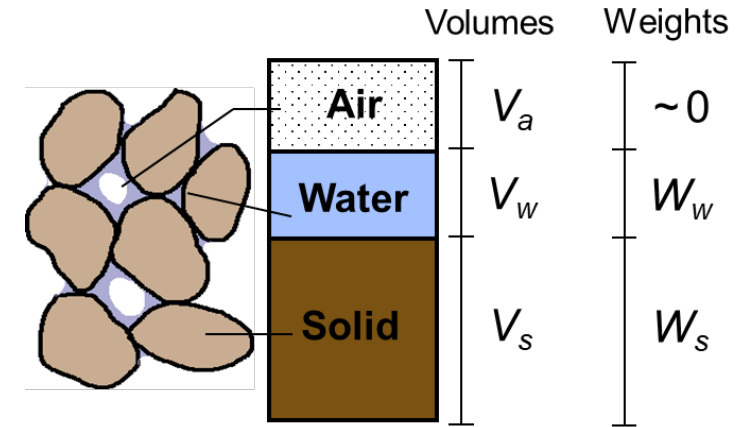
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Three-phase description

A complete description of the rheological behaviour of unsaturated soils requires

- Soil water retention behaviour
- Stress-strain behaviour

How S_r changes for a given suction ?

How the soil deforms?

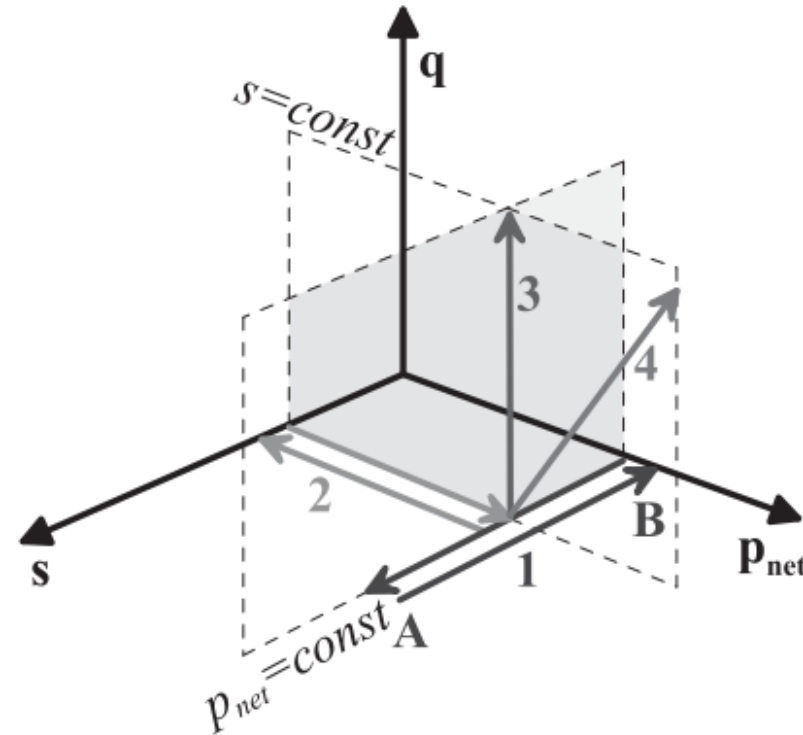
Stress variables

Work-conjugate variables

Stress-strain variables for constitutive modelling of unsaturated soils

$$\begin{pmatrix} \sigma' \\ S \end{pmatrix} = \begin{pmatrix} \sigma_{net,ij} + S_r \cdot s \delta_{ij} \\ S_r \end{pmatrix} \quad \begin{pmatrix} \varepsilon \\ S_r \end{pmatrix}$$

Possible stress paths



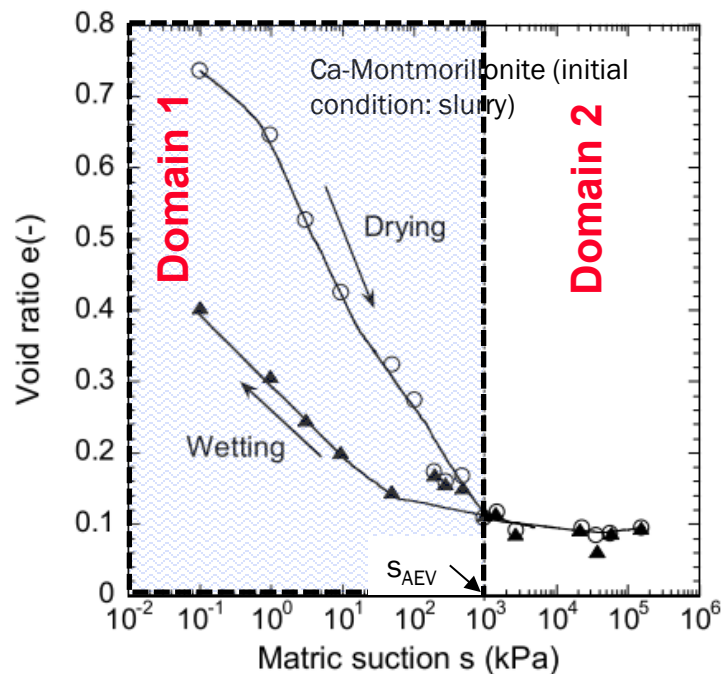
The possible stress paths to which a soil can be subjected can be highlighted in the 3D plane (s, p_{net}, q)

Volumetric behaviour

Isotropic hydraulic loading (the suction is increased at a constant net stress)

Plane void ratio - suction

The **strains** induced by a drying or wetting process can be **elastic or elasto-plastic**. For some materials (usually if prepared from a slurry), we observe the following:

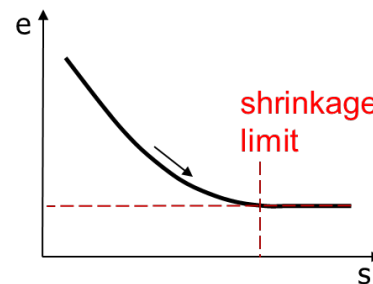


DOMAIN 1 – Elasto-plastic strains

If the suction is increased at a value lower than the air entry value and a wetting is performed, **NOT all the change in volume is recovered**

DOMAIN 2 – Elastic strains

If the suction is increased at a value higher than the air entry value and a wetting is performed, the **volume change** induced in the range of suction after the air entry value **is recovered**



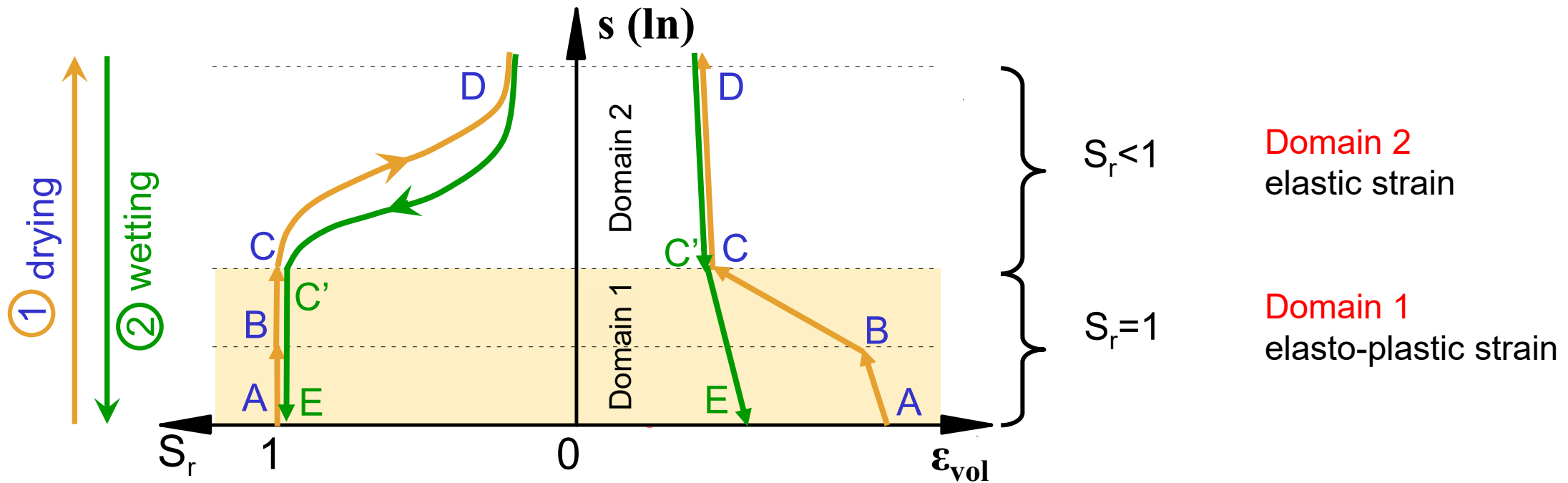
Shrinkage limit: value of suction starting from which no more significative variation in volume associated to an increase in suction are observed.

Volumetric behaviour

Isotropic hydraulic loading (the suction is increased at a constant net stress)

Plane suction-degree of saturation and suction-volumetric strain

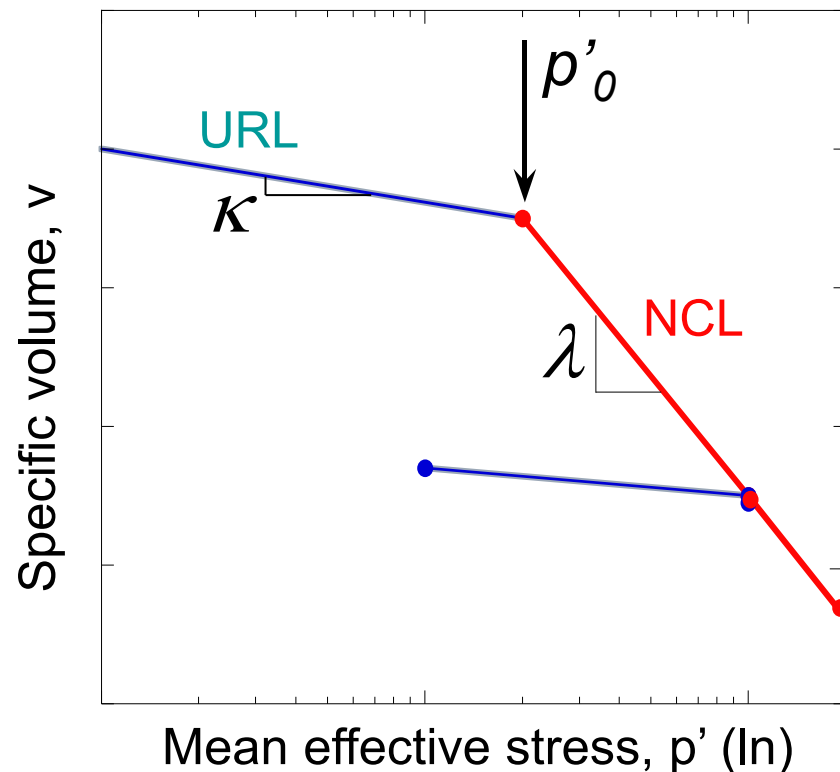
The strains induced by a drying or wetting process can be elastic or elasto-plastic. For some materials (usually if prepared from a slurry), we observe the following:



Volumetric behaviour

Isotropic mechanical loading – RECALL of the main equations and variables

Plane specific volume-mean effective stress



Normal Compression Line (NCL)

$$v = N - \lambda \ln p'$$

Unloading Reloading Line (URL)

$$v = v_k - k \ln p'$$

λ : slope of the NCL (plastic compressibility)

κ : slope of the URL (elastic compressibility)

N : specific volume at $p'=1$ in the adopted measurement system

v_k : specific volume at $p'=1$ in the adopted measurement system

p'_0 : Preconsolidation pressure

Volumetric behaviour

Isotropic mechanical loading at constant suction

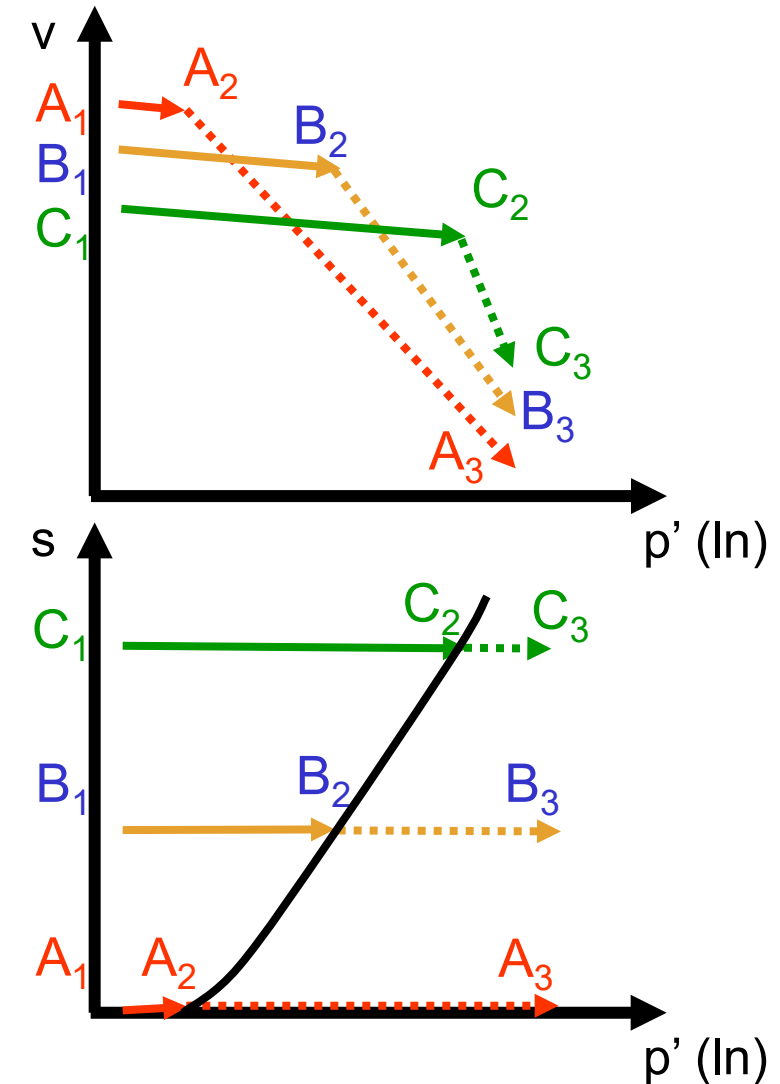
Plane specific volume-mean effective stress

An increase of suction implies:

- an increase of the preconsolidation pressure (apparent hardening)
- a variation of the plastic compressibility
- elastic compressibility remains constant

Points A_2 , B_2 and C_2 delimit the elastic domain for each path

The curve connecting all the preconsolidation pressure at the different suction values is the yield locus in the plane (s - p'). It is called **Loading-Collapse curve (LC)**.

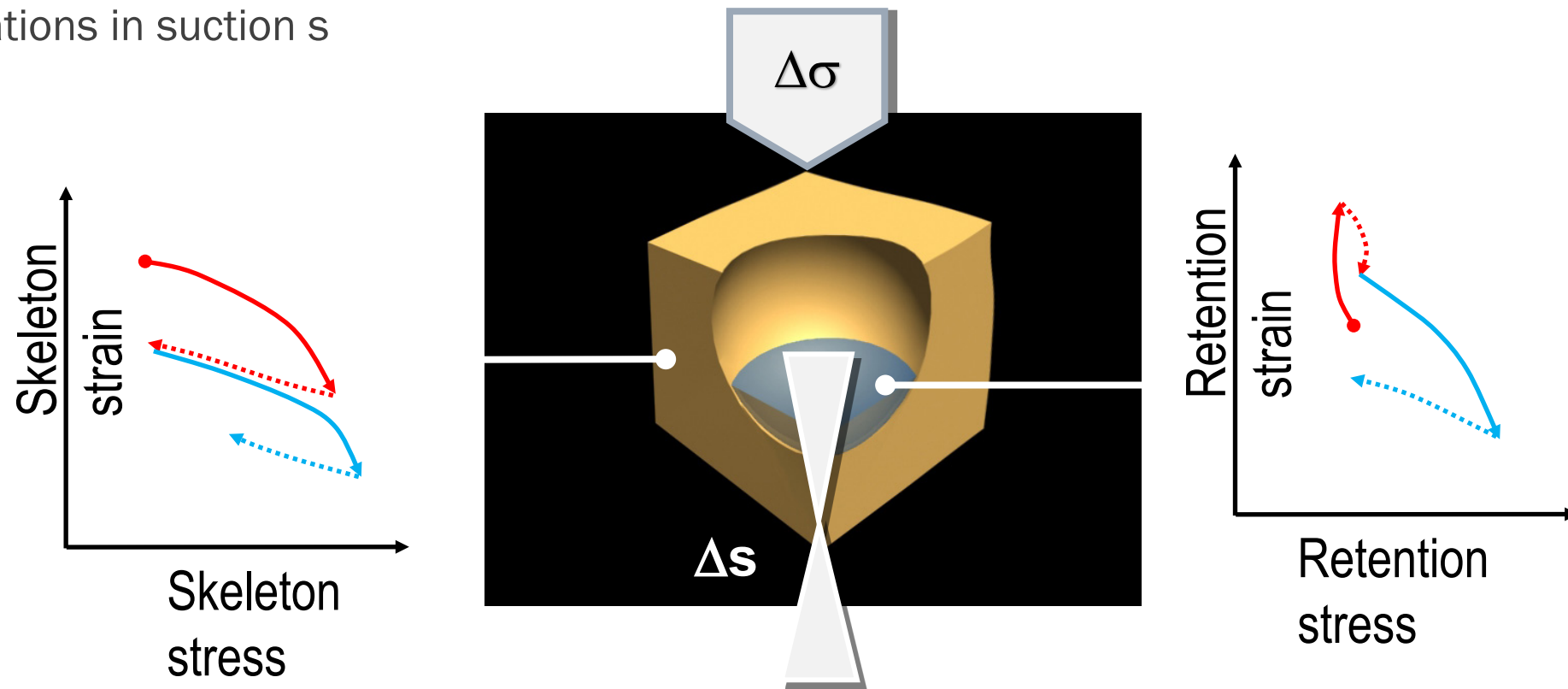


Volumetric behaviour

Skeleton straining and **saturation changes** are a response to

(a) External mechanical loading σ

(b) Variations in suction s

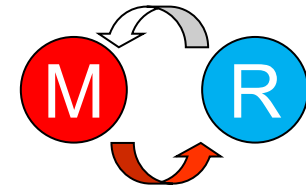
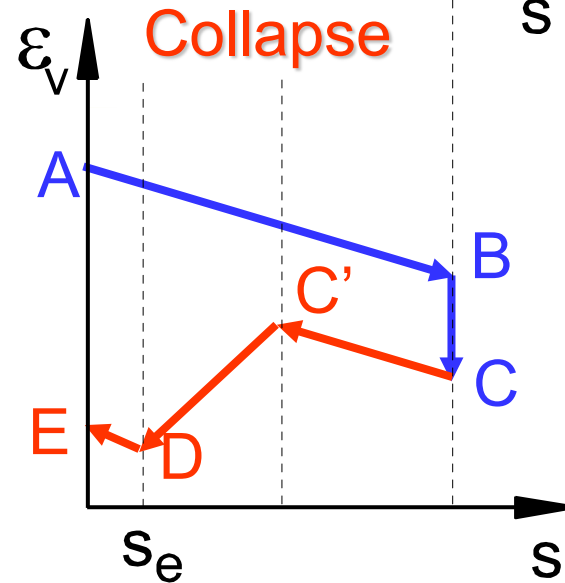
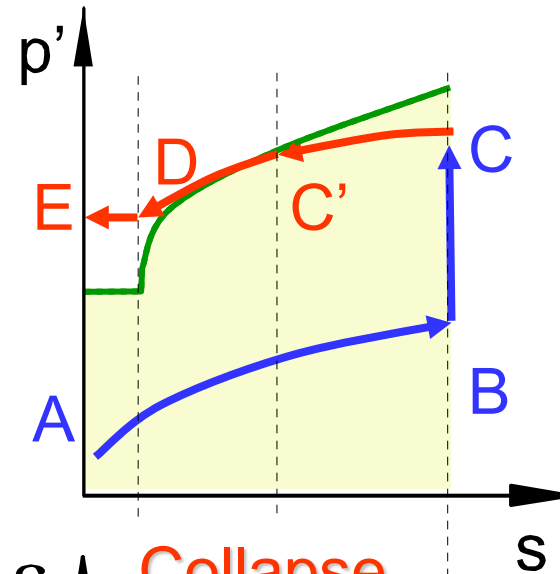


Irreversible processes in both cases

Wetting collapse

- **AB: drying** ($p = \text{const.}$)
- **BC: mechanical consolidation** ($s = \text{const.}$)
- **CC'**: wetting – elastic **swelling**
- **C'D**: wetting – **plastic collapse**
- **DE**: wetting – elastic **swelling**

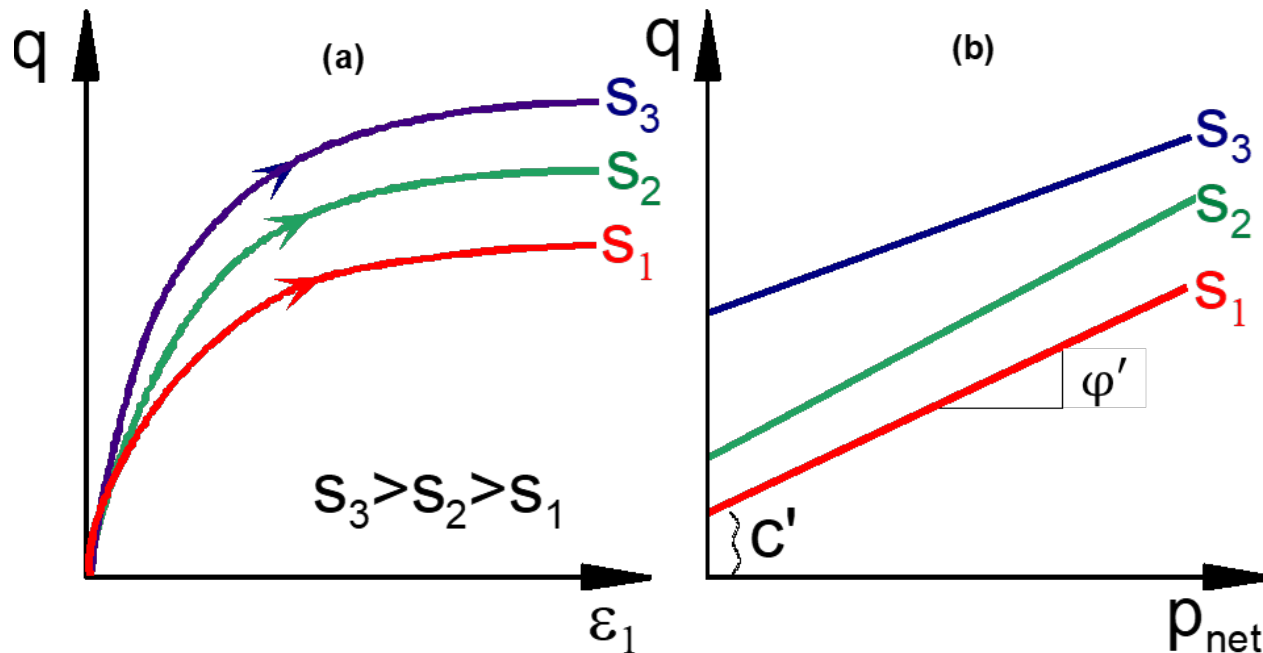
— LC curve Elastic domain



Shear strength

Deviatoric mechanical loading at constant suction

Planes $q-\varepsilon_1$ and $q-p_{\text{net}}$



Effects of suction on the shear strength

An increase of suction implies

- an increase of the cohesion intercept c'
- a variation of the shear strength angle φ'
- an increase of the overall shear strength

Shear strength

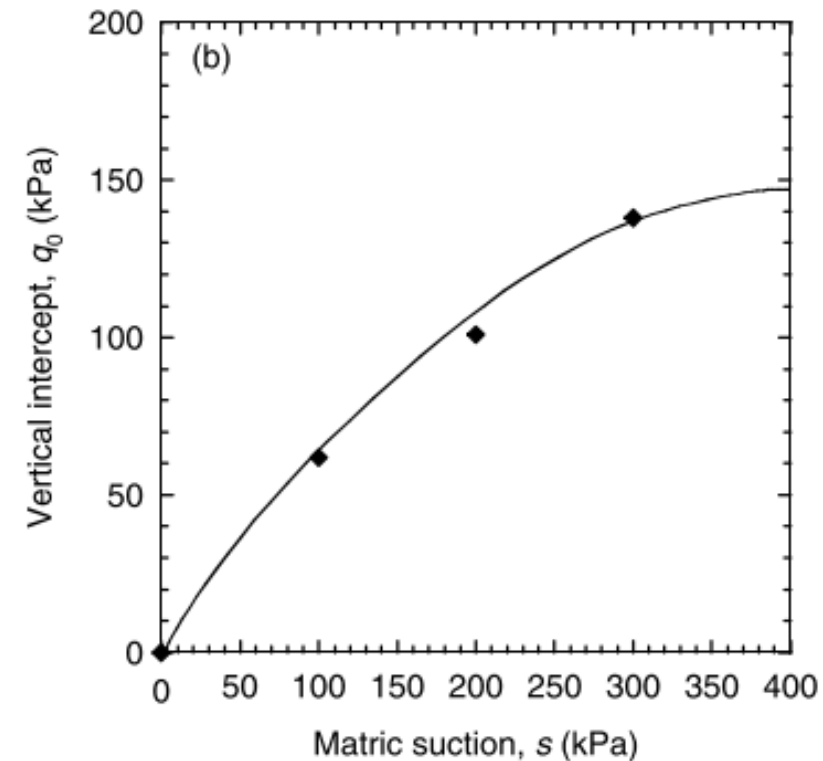
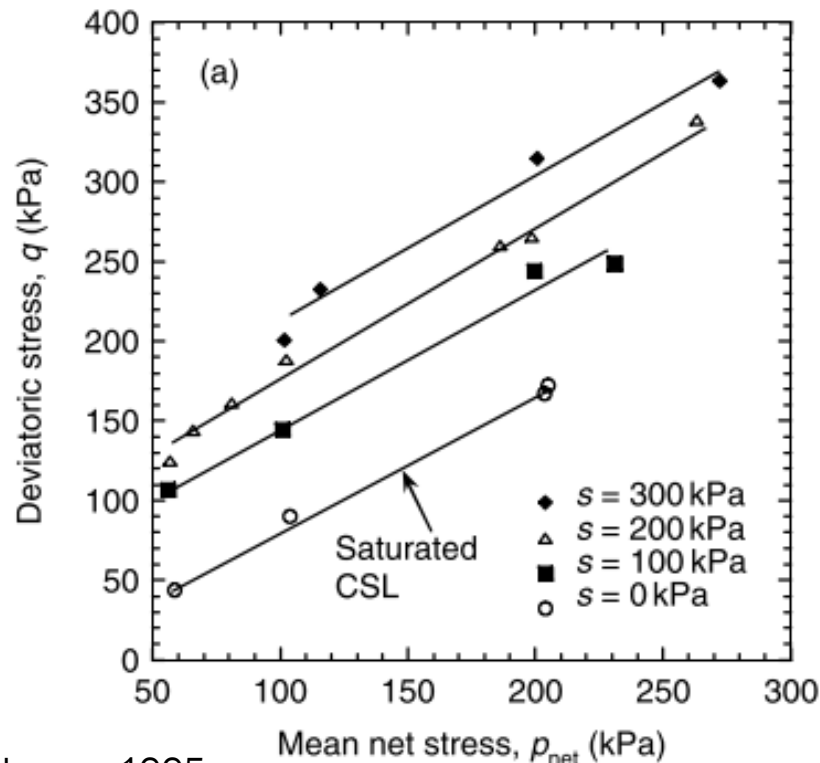
Deviatoric mechanical loading at constant suction

Planes q - p_{net}

q_0 as function of suction

Effects of suction on the shear strength

- *Apparent* cohesion is suction dependent
- Friction angle remains fairly constant
- Increase of the overall shear strength



Data from Wheeler & Sivakumar, 1995

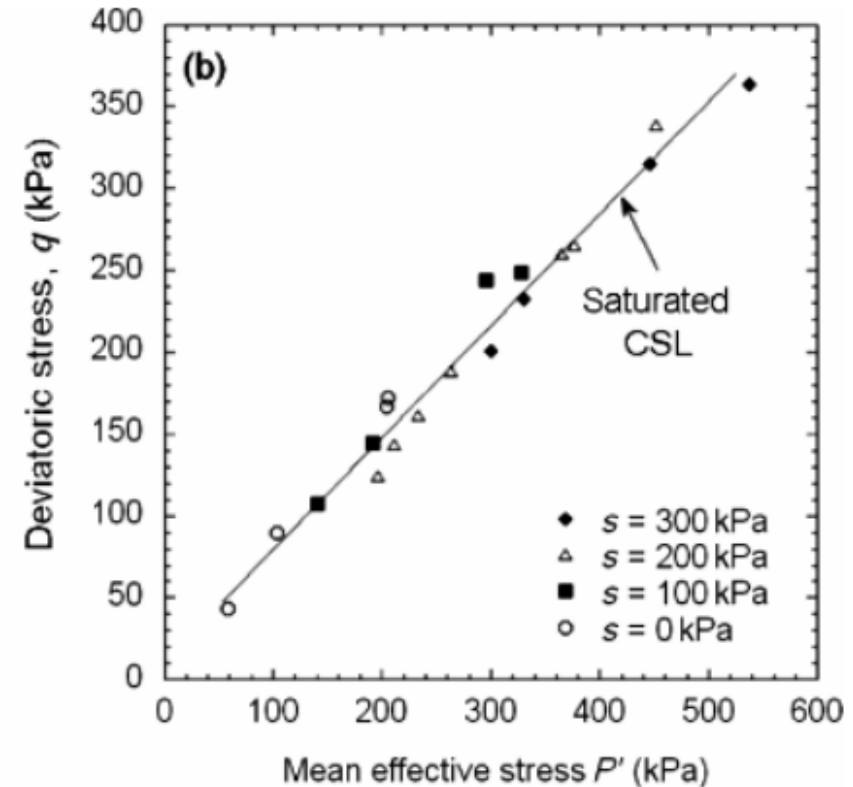
Shear strength

Deviatoric mechanical loading at constant suction

Planes q-p'

$$p' = p_{net} + S_r s$$

$$p_{net} = p - p_a \quad p = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$



Data from Sivakumar, 1995

The adoption of a **suitable definition of effective stress** allows defining a **unique shear strength envelope** irrespective of the degree of saturation of the soil.

An increase of the **mean effective stress**, related to an increase in **suction** and/or an increase of the **mechanical load**, results in an increase of the shear strength.

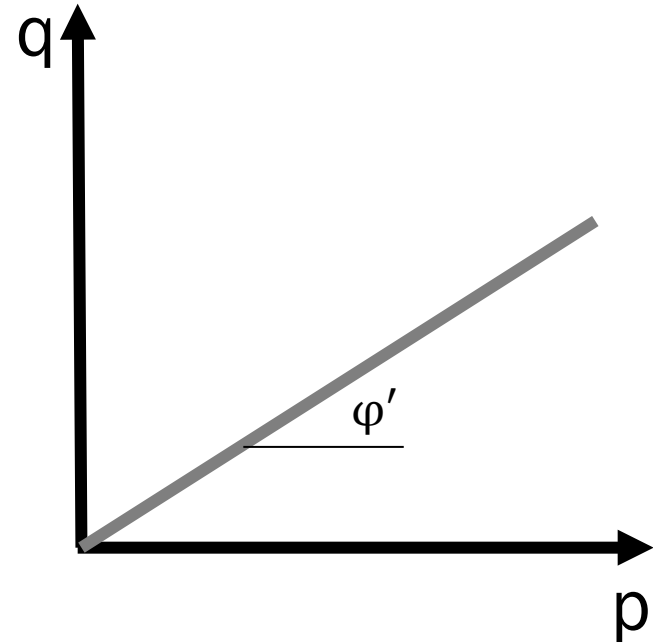
Shear strength

Deviatoric mechanical loading at constant suction

Planes q-p'

$$p' = p_{net} + S_r s$$

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The adoption of a **suitable definition of effective stress** allows defining a **unique shear strength envelope** irrespective of the degree of saturation of the soil.

An increase of the mean effective stress, related to an increase in **suction** and/or an increase of the **mechanical load**, results in an increase of the shear strength

Conclusion

- **Unsaturated soils** are characterized by **two fluids and a solid phase**
- The **negative pore water pressure** is usually expressed in terms of **suction**
- The **water retention curve** describes the relationship between degree of saturation and suction
- The **Darcy's law is still valid** for unsaturated soils, the permeability is expressed in function of the degree of saturation
- The **Terzaghi's effective stress** is still valid if the pore water pressure is negative and the **degree of saturation is close to 1**. Otherwise, a more **general expression** has to be used
- The **volumetric behaviour and the shear strength** of a soil are affected by variation in suction

Thank you for your attention

